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Third Semester B.E. Degree Examination, January 2013
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting atleast TWO questions from each part.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. 2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. For any three sets A, B, C prove the following :
i) $A \cap (B - C) = (A \cap B) - C$
ii) $(A - B) \cap (A - C) = A - (B \cup C)$ **(06 Marks)**

b. Using Venn diagram, prove that, for any three sets A, B, C
$A \Delta (B \Delta C) = (A \Delta B) \Delta C$ **(06 Marks)**

c. A fair die is thrown (tossed) twice. Find the probability that
i) Even numbers occur on both throws
ii) An even number occurs in atleast one throw. **(08 Marks)**

2 a. Define tautology and contradiction. Prove that, for any propostions p, q, r, the compound proposition
$[(p \to q) \wedge (q \to r)] \to (p \to r)$. **(08 Marks)**

b. Verify the principle of duality for the logical equivalence :
$[\sim (p \wedge q) \to \sim p \vee (\sim p \vee q)] \Leftrightarrow (\sim p \vee q)$. **(06 Marks)**

c. Define the following :
i) Rule of syllogism ii) Modus pones iii) Modus Tollens. Test whether the following argument is valid : if Sachin hits a century, he gets a free car. Sachin does not get a free car.
∴ Sachin has not hit a century. **(06 Marks)**

3 a. Define : i) Open sentence ii) Quantiziers. Write down the following propositions in symbolic form and find its negation :
"If all triangles are right – angled, then no triangle is equiangular". **(07 Marks)**

b. Find whether the following argument is valid : no engineering student of first and second semester studies logic.
Anil is an engineering student who studies logic.
∴ Anil is not in second semester **(07 Marks)**

c. Give : i) A direct proof ii) an indirect proof and iii) proof by contradiction, for the following statement. "If n is an odd integer, then n+11 is an even integer. **(06 Marks)**

4 a. Prove that, for each $n \in z^+$
$1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = \frac{1}{6} n (n + 1) (2n + 1)$. **(06 Marks)**

b. A sequence $\{a_n\}$ is defined recursively by
$a_1 = 4$, $a_n = a_{n-1} + n$ for $n \geq 2$. Find a_n in explicit form. **(06 Marks)**

c. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, prove that
$\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$ for all positive integers n. **(08 Marks)**

PART – B

5 a. Define Cartesian product of two sets. For any three non – empty sets A, B, C prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(06 Marks)**

b. Prove that a function $f : A \rightarrow B$ is invertible if and only if it is one – to – one and onto. **(06 Marks)**

c. Define stirling number of the second kind. Find the number of ways of distributing four distinct objects among three identical containers, with some containers possibly empty. **(08 Marks)**

6 a. For a fixed integer $n > 1$, prove that the relation "congruent modulo n" is an equivalence relation on the set of all integers, Z. **(08 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

let A, B, C, D be sets. Suppose R is a relation from A to B, S is a relation from B to C and T is a relation from C to D. Then prove that Ro(SoT) = (RoS) oT **(06 Marks)**

7 a. Define an Abelian group. List G be the set of all non – zero real numbers and let $a * b = \frac{1}{2} ab$ in G. Show that (G, ∗) is an Abelian group. **(08 Marks)**

b. Define a subgroup. Let G be a group and let $J = \{x \in G / xy = yx \text{ for all } y \in G\}$. Prove that J is a subgroup of G. **(06 Marks)**

c. State and prove Lagrange's theorem. **(06 Marks)**

8 a. Define a ring. If R is a ring with unity and a, b are units in R, prove that ab is a unit in R and that $(ab)^{-1} = b^{-1}a^{-1}$. **(06 Marks)**

b. If $f : G \rightarrow H$, $g : H \rightarrow K$ are homomorphisms, prove that the composite function $gof : G \rightarrow K$, where $(gof)(x) = g\{f(x)\}$, is a homomorphism. **(06 Marks)**

c. Define a group code. Let $E : Z_2^m \rightarrow Z_n^2$ be an encoding function given by a generator matrix G or the associated parity – check matrix H. them prove that $C = E(Z_2^m)$ is a group code. **(08 Marks)**

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